Learning and Instruction 17 (2007) 322–335

Learning and Instruction

www.elsevier.com/locate/learninstruc

What kinds of perceptions and daily learning behaviors promote students' use of diagrams in mathematics problem solving?

Yuri Uesaka [a,*], Emmanuel Manalo [b], Shin'ichi Ichikawa [c]

[a] *Department of Educational Psychology, Graduate School of Education, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan*

[b] *The Student Learning Centre, The University of Auckland, New Zealand*

[c] *Department of Educational Psychology, Graduate School of Education, The University of Tokyo, Japan*

Abstract

This study investigated factors promoting the use of self-constructed diagrams by examining students' perceptions and daily class activities, and comparing Japanese ($n = 291$) and New Zealand ($n = 323$) students. Algebra word problems and a questionnaire were administered. The results revealed that the New Zealand students used diagrams more often and scored higher than their Japanese counterparts. Lack of confidence and perceptions of difficulty in diagram use, and viewing diagrams more as a strategy that teachers use, were found to link with lower use. Possible ways of promoting diagram use in math word problem solving are discussed.

Keywords: Math word problem solving; Self-constructed diagrams; Cross-national comparison; Student strategy use

1. Introduction

1.1. Math word problem solving and the importance of diagram use

In mathematics education, the development of students' abilities in solving math word problems has been an important topic (see, e.g., Polya, 1945; Reed, 1999; Schoenfeld, 1985, 1992). Some researchers have described solving math word problems as one of the most difficult areas for students (e.g., Reed, 1999; Yoshida, 1991). Standards published by the National Council of Teachers of Mathematics in the US (NCTM, 2000) and the National Curriculum of Japan published by the Japanese Ministry of Education (1998) both focus on the importance of learning tasks relating to real-world situations – the very area that the majority of math word problems deal with.

As noted, solving math word problems is not easy for many students and, because of this, many studies have looked into and proposed ways for overcoming the difficulties that some students encounter. One approach is to cultivate students' internal resources – in other words, to develop their knowledge of heuristics and skills in strategies use

* Corresponding author. Tel.: +81 3 5841 3923; fax: +81 3 5841 3919.
E-mail address: yuesaka@educhan.p.u-tokyo.ac.jp (Y. Uesaka).

0959-4752/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.learninstruc.2007.02.006

(Collins, Brown, & Newman, 1989; De Corte, Greer, & Verschaffel, 1996; Polya, 1945; Schoenfeld, 1985, 1992). Polya (1945), one of the earliest in promoting this approach, emphasized the importance of cultivating students' abilities in using heuristics, which includes promoting the use of diagrams and reminding students about methods used in other similar problems. Later studies (e.g., Schoenfeld, 1985) have confirmed the effectiveness of heuristics and strategies use.

Among the many strategies that have been suggested to improve efficacy in solving math word problems, using diagrams has been described as one of the most effective. For example, Hembree (1992) demonstrated by employing meta-analysis that using diagrams was the most efficient among strategies that had been suggested as helpful for problem solving. Math teachers' frequent use of diagrams in class (Dufour-Janiver, Bednarz, & Belanger, 1987) is another strong indicator of the value of diagrams to their users.

Numerous studies on the use of diagrams have attempted to explain their contribution to efficiency in problem solving. For example, Larkin and Simon (1987) argued that diagrammatic representation is computationally more efficient than sentential representation because it minimizes labels, gathers related information in one place, and facilitates easier recognition of the situation with the help of the visual system. Other studies have empirically demonstrated the beneficial effects of providing a specific diagram or visual representations in problem solving (e.g., Ainsworth & Th Loizou, 2003; Cheng, 2004; Mayer, 2003; Pedone, Hummel, & Holyoak, 2001).

Although the results of some studies have suggested that self-constructed diagrams are not always effective (e.g., De Bock, Verschaffel, Janssens, Van Dooren, & Claes, 2003; Van Essen & Hamaker, 1990), some recent studies have also shown that self-constructed diagrams are powerful heuristics in problem solving situations (e.g., Cheng, 2002; Koedinger & Terao, 2002; Stern, Aprea, & Ebner, 2003; see also a review by Cox, 1999). For example, Stern et al. (2003) found that actively constructing and using linear graphs as reasoning tools while learning economics can better facilitate a transfer effect across subject content compared to a condition of receptive diagram use (i.e., where learners receive the diagrams to use rather than constructing their own).

1.2. Lack of spontaneity in students' use of diagrams

Many school students do not share the positive view of diagram use that teachers and researchers tend to hold. Some studies have pointed out that few students use diagrams spontaneously when they encounter difficult problems even though their teachers often use diagrams when explaining how to solve problems (e.g., Dufour-Janiver et al., 1987; Ichikawa, 1993, 2000). This lack of spontaneous use could be due to students not knowing how to construct diagrams, as well as to not appreciating the potential benefits of using diagrams. Ichikawa (1993), for example, reported a case of an eighth grade girl who tried to solve a problem without the use of any diagram and gave up solving it (even though she was previously provided instruction and encouraged by the researcher to use diagrams). Subsequently, however, the girl was able to solve the problem easily after the researcher urged her to use diagrams. Ichikawa concluded from this case that students do not use diagrams spontaneously if they do not perceive the efficiency that results from their use — even if they are perfectly capable of using diagrams well.

Students' spontaneous diagram use in math word problem solving also appears to be influenced by the story context that comes with the problem: Hall, Kibler, Wenger, and Truxaw (1989) found that students used diagrams more often when attempting to solve distance-related problems compared to less spatial problems. Uesaka (2003) further found that the influence of the story context on diagram use was mediated by the structure of the problem (cf. Mayer, 1981) — in other words, whether the problem involved "one-object" or "two-objects" where, in the former, one-object or subject does the work, while two-objects or subjects do the work in the latter. These findings therefore provide further insight into factors that influence diagram use. However, they deal with factors that are external to the student and thus fail to explain individual differences in the spontaneity with which students use diagrams.

Although few studies have examined the factors behind individual differences in spontaneous diagrams use, the kinds of factors that influence decisions to use particular strategies for more effective learning have been explored in numerous studies on learning strategy. For example, it has been found that the perception of the likely efficiency that will result from the use of a strategy is one of the important factors that contribute to the decision to use the strategy (McCombs, 1988; Sato, 1998; Uesaka, 2002). Self-efficacy in learning (Pintrich & De Groot, 1990; Zimmerman & Martinez-Pons, 1990), and the perception of strategy cost (McCombs, 1988; Sato, 1998) have also been identified as factors relating to strategy use.

Most of the previous studies on students' strategy use have focused on cognitive factors that influence the decision to use these strategies. Very few studies have looked at behavioral factors that influence students' decision making in this area. It is clear, however, that the use of diagrams might be decided not only by cognitive factors but also by students' daily learning behaviors for developing their abilities to use the relevant skills, and teachers' behaviors such as the provision of encouragement. Even if students perceived the efficiency benefit of diagram use, they may not be able to use diagrams effectively if, for example, they do not have adequate practice in using diagrams in class or at home. Thus, it is equally important to explore aspects of students' behaviors, and their perception of their teachers' behaviors, that could contribute to their developing skills in diagram use.

1.3. Comparison of students in Japan and New Zealand

The present investigation sought to find out the factors that promote diagram use by gathering data from students in two countries: Japan and New Zealand. New Zealand students were predicted to be more active in diagram use compared to their Japanese counterparts based on the following three reasons.

The first reason relates to the national mathematics curricula of these two countries: these are quite different in the points they make about diagrams use. The national curriculum in New Zealand not only stresses the importance of teaching how to understand diagrams, but also considers the use diagrams as a communication tool as one of the crucial goals of algebra. It declares that ''The mathematics curriculum intended by this statement will provide the opportunities for students to develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalizations'' (New Zealand Ministry of Education, 1992, p. 10). In contrast, the Japanese national curriculum emphasises understanding diagrams and targeted mathematical concepts, but it does not include the importance of using diagrams as tools for problem solving and communication. This difference in the national curricula might therefore make it more likely for students in New Zealand to view diagrams as tools for problem solving and to use them more spontaneously when problem solving.

The second reason comes from the performance of students from the two countries in PISA (Program for International Student Assessment), a project commissioned by the Organization for Economic Co-operation and Development (OECD) and which assessed the knowledge and skills of 15-year-old students from the participating industrialized countries (OECD, 2004). Both countries were ranked very high in PISA 2000 and PISA 2003. However, in PISA 2003, New Zealand students scored above average in the section on strategies use, but Japanese students were ranked as one of the lowest (40th among 41 countries), suggesting a difference in how students from the two countries approach problem solving.

The third reason is derived from reported observations of high percentages of ''white paper'' (i.e., no answers provided or attempted) and ''bland answers'' in Japanese students' scripts in international assessment test such as PISA 2003 and TIMSS (Trends in International Mathematics and Science Study) 2003, which were pointed out in a detailed report published by the National Institute for Educational Policy Research in Japan (Nagasaki, 2005). Although the results of TIMSS 2003 (Mullis, Martin, Gonzalez, & Chrostowski, 2004) suggest that Japanese teachers spend a lot of time interpreting diagrams in junior high school math classes, the results also suggest that Japanese students give up too easily when using a trial-and-error process in solving pen and paper problems, and that the use of external resources for problem solving is a point of weakness for them.

If the data collected in the present study support the prediction of greater diagram use among New Zealand students, a good starting point would be established for considering some of the wider issues that could influence students' predispositions toward the use of diagrams when attempting to solve math word problems – including the perceptions and daily learning behaviors that might be associated with these.

1.4. Research questions and outline of steps taken

The main research question of this study was: What kinds of perceptions and daily learning behaviors are related to the actual use of diagrams in math word problem solving? In order to address this question, four steps were taken. First, a comparison was made of the spontaneous use of diagrams among students in Japan and New Zealand – thus generating a subquestion of: Do Japanese and New Zealand students differ in the extent to which they spontaneously use diagrams in solving math word problems? As noted, New Zealand students were predicted to show greater use of diagrams compared to their Japanese counterparts, and it was expected that this would be evident in all the problems provided irrespective of story context and structure.

Second, if the analysis confirmed the predicted differences between the two countries, an analysis of the students' responses would be conducted to identify the kinds of perceptions and daily learning behaviors in which the students from the two countries might differ. A second subquestion would then be: In what ways do the Japanese and New Zealand students differ in their reported perceptions and daily learning behaviors relating to math word problem solving and diagram use?

However, even if some differences in the students' perceptions and daily learning behaviors are found, it would not be immediately clear whether those differences are related to actual diagram use. Thus, thirdly, the correspondence and correlations between students' responses to the questionnaire items and their actual use of diagrams would be analysed to examine whether any identified differences between the two countries are really related to the spontaneous use of diagrams. This analysis would address the subquestion: Are the identified differences between the Japanese and New Zealand students in their reported perceptions and daily learning behaviors related to actual diagram use as evidenced in their problem solving performance in this study? Finally, and on the basis of the findings that are obtained, the possible reasons for students' poor use of diagrams in math problem solving would be reconsidered and strategies that could be employed for overcoming such tendencies would be discussed.

2. Method

2.1. Participants

The participants were 614 secondary school students from Japan and New Zealand, aged 13–15 years old (mean ages: Japan = 13.28 years, New Zealand = 13.97 years). The Japanese cohort ($n = 291$; female = 131, male = 160) comprised of students from four junior high schools in the Kanto area (which includes Tokyo) in Japan, while the New Zealand cohort ($n = 323$; female = 134, male = 189) comprised of students from five secondary schools in the upper half of the North Island of New Zealand (which includes Auckland).

In both Japan and New Zealand, every effort was made to ensure that the range in students' abilities in mathematics was represented in the student groups included in the study. In both countries, data were collected from schools representing the full range of the ''ability'' spectrum, from what are considered ''high level'' schools through to those that are considered ''lower level'' schools. In Japan, although ''ability'' differences between schools are generally considered to be small, schools included in the study were from both central city locations and outlying regions. In New Zealand, schools have decile ratings that provide an approximation not only of the general abilities of the students who attend, but also their socio-economic backgrounds (cf. New Zealand Ministry of Education, 2005). Both a top decile school and a bottom decile school were included in this study, as well as three schools in between. In addition to the school selection, the math teachers in the schools who participated were requested to administer the problems and questionnaires to approximately equal numbers of ''high,'' ''average,'' and ''low'' ability math classes.

2.2. Procedure

A booklet containing math word problems and questionnaires were given to the participants during their regular math class. In Japan, the booklet was written in Japanese, while in New Zealand it was written in English. (The equivalence of the English and Japanese versions of the booklet was ensured partly through extensive consultations with bilingual colleagues and math teachers during the translation process from Japanese to English. The equivalence of the versions was further checked and confirmed by two professional colleagues of the authors – one in Japan and one in New Zealand – who are fully fluent in both languages, and have no vested interest in the outcomes of the study.) The participants were told that the purpose of the research was to find out how they solved math word problems, and they were requested to show their working throughout.

The first part of the booklet contained two problems (problem 1 and problem 2), one problem to a page with ample space below each problem for the participants to work out the solution and provide their answer. The participants were told that they had 4 min to work on each math problem, and not to go to the next problem/page until instructed to do so. The two problems were then followed by a questionnaire concerning the participants' views about the use of diagrams in solving math word problems (these questionnaire items are grouped under the headings of ''Views about the usefulness of diagrams'' and ''Perceptions of the difficulty of diagram use'' in Tables 6 and 7). The questions all required responses on a five-point Likert scale, with the end points labelled (i.e., ''not at all'' for 1, and ''definitely'' for 5).

The questionnaire section was then followed by two more math word problems (problems 3 and 4) isomorphic to the first two, but with instruction to use diagrams. However, analyses of the participants' responses to these latter two problems are not dealt with in the present paper. Following these problems was another questionnaire which solicits information about the participants' daily learning behaviors, perceptions about their teachers' use of diagrams, views about math and their performance in class (these questionnaire items are grouped under the headings of ''Daily learning behaviors'' and ''Perceptions of teacher's behaviors'' in Tables 6 and 7), as well as demographic details (gender, age, etc.). On the items concerning daily learning behaviors, the participants were asked not about their behavior during the time of their participation in the present study, but about what they usually do in everyday situations. Most of the questions again required responses on a five-point Likert scale. Administration of the problems and questionnaires booklet took approximately 30 min in total.

Two versions of the booklet were made (differing only in the story context attached to the math word problems, see explanation under *Materials* below) and, in each class, approximately half of the participants were randomly given one version of the booklet while the other half were given the other version. One version of the booklet contained math word problems involving length, and the other version contained non-length problems. In Japan, 149 students received the booklet with length problems, and 142 received the booklets with non-length problems. The corresponding numbers that received each kind of booklet in New Zealand were 173 and 150 students, respectively.

2.3. Materials

To ensure that the problems used in this study captured the variety that students have to deal with in real life (noted in Section 1), different kinds of problems were prepared. Apart from the length and non-length distinctions described above, in each version of the booklet, two categories of problems were included: one-object problems and two-object problems (similar to ones used in Uesaka, 2003). Examples of the different kinds and categories of math word problems used in the present study (those actually used for problems 1 and 2 in the two versions of the booklet) are shown in Table 1.

The questionnaire assessing students' perceptions about the use of diagrams employed in this study was originally constructed by Uesaka (2002) based on a 'bottom-up' research approach she had used to examine students' reasons for using or not using diagrams in math word problem solving. This questionnaire consisted of two sections: the first

Table 1
Examples of the kinds and categories of math word problems used

Kind	Category	
	One-object	Two-object
Length	You light a candle and it starts to burn. It burns at a constant rate, which means that it burns at the same speed throughout. After 5 min, the candle is 10 cm in length. After 7 min, it is 6 cm. How long does it take for the candle to burn out?	Tom's house and Hannah's house are connected by one road which is 600 m long. Tom and Hannah talked on the telephone and decided to play with each other. They leave their own house at the same time and start to walk toward the other's house. They meet on the road 5 min later. The place where they meet is 100 m closer to Hannah's house than the half-way point. How fast did Tom walk (per minute)? How fast did Hannah walk (per minute)?
Non-Length	A mouse starts to eat a piece of cheese. The mouse eats at a constant rate, which means it eats at the same speed throughout. After 5 min, there are 10 g left of the cheese. After 7 min, there are 6 g left. How long does it take for the mouse to finish eating the piece of cheese?	There is a pond that can contain up to 600 L of water. Now the pond is empty; so using two taps, A and B, you start to fill the pond with water. Water comes out of both taps at a constant rate. After 5 min, the pond is full. From tap A came 100 L more water than half of all the water in the pond. How fast did water come out of tap A (per minute)? How fast did water come out of tap B (per minute)?

assessed students' perceptions of efficiency resulting from diagram use, and the second their perceptions of difficulties associated with diagram use.

Items included in the questionnaire about students' daily learning behaviors and students' perceptions of their teachers' behaviors were specifically developed for this study as the authors could not find any existing questionnaires that appropriately examined these. Development of the items was based on interviews with, and suggestions from, math teachers.

2.4. Scoring

The first author scored the students' performance in solving the math word problems, determining whether each of the answers they provided was correct (P+) or incorrect (P−). In addition, she assessed the students' use of diagrams for each of the problems, determining in each case whether at least one diagram was used (D+) or no diagram was used at all (D−). For the purposes of the present study, a diagram was defined as any representation of the problem other than words (on their own), sentences, or numerical formulas. Tables were counted as diagrams and, for the purposes of this study, a table was defined as a depiction of at least a pair of values arrayed to represent two related variables. Drawings or illustrations deemed unrelated to the problem were categorized as D−. A colleague of the first author's was also employed as an independent assessor of diagram use in the students' work. The inter-rater agreement was found to be 96.3%, which the present authors deemed as satisfactory. Examples of diagrams produced by participants are shown in Fig. 1.

The qualities of the diagrams constructed by the students were also analysed to check for any differences between the two countries. Both the structure and the information contained in the students' diagrams were assessed in determining their quality (see Table 2). Where the structure was concerned, a diagram was placed in the higher "category A" if it represented the situation correctly. Conversely, if a diagram was deemed not to represent the situation correctly, it was placed in the lower "category B". Where the information contained in the diagrams was concerned, both the amount and the kinds of relevant information were considered. Thus, for example, diagrams were placed in the highest "category A" if they contained additional inferences drawn from the problem given, but they were placed in the next "category B" if they contained all the numbers specified in the problem but without evidence of additional inferences. At the other end of the scale, diagrams were placed in the lowest "category E" if they contained no numbers at all.

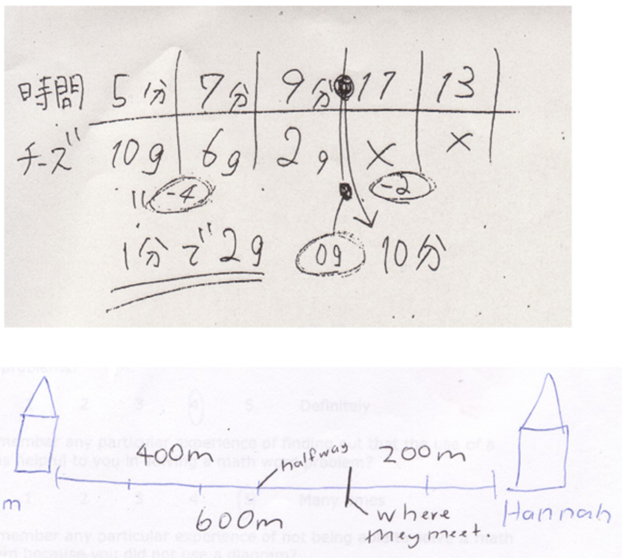

Fig. 1. Examples of diagrams produced by student participants in solving a one-object problem (top, Japanese student) and a two-object problem (bottom, New Zealand student).

Table 2
The criteria used for evaluating the quality of diagrams students produced for the one- and two-object problems

Points of evaluation	Categories	Criteria for placement in the categories
Structure of diagram	A	Represents the situation correctly
	B	Does not represent the situation correctly
Information contained in diagram	A	Contains additional inferences drawn from the problem
	B	Contains all of the numbers specified in the problem
	C	Contains some of the numbers specified in the problem
	D	Contains some numbers, but all of them are incorrect or unrelated to the problem
	E	Contains no numbers

Using this method of categorization, the students' diagrams were divided into those deemed to be of ''high quality'' and those of ''low quality''. Diagrams were classified as being high in quality if they represented the situation of the problem correctly and they contained more relevant information. More specifically, a diagram produced for the one-object problems was classified as high quality if it had been placed in category A for both structure and content. A diagram produced for the two-object problems, on the other hand, was classified as being high quality if it had been placed in category A in structure and at least category B for content. The decision on where to place the cut-off point for content was made partly in consideration of the numbers of diagrams falling into each of the high-quality and low-quality categories: a considerably higher percentage of diagrams produced for the one-object problems contained appropriate inferences, which was not the case where diagrams produced for the two-object problems were concerned – thus only diagrams that were categorized as A for content were classified as high quality where the one-object problems were concerned.

3. Results

3.1. Student problem solving performance and diagram use

Table 3 shows the percentages of correct answers to the math word problems provided by the Japanese and New Zealand students, according to the kinds and categories of problems given. In all cases, the percentages of correct responses provided by the New Zealand students were significantly higher than the corresponding figures from the Japanese students.

The students' diagram use in relation to the problems given was analysed using a log linear model. In this analysis, the students were grouped according to their diagram use (i.e., used none, used only in the first one-object problem, used only in the second two-object problem, and used in both problems), with country and story context of problems given (i.e., length or non-length) as between-subject factors. A main effect of response type was found ($\chi^2_{(1)} = 4.31$, $p < .05$), indicating that, overall, the proportion of students who did not use a diagram at all was the greatest. Further, an interaction effect between the response type and story context was found ($\chi^2_{(1)} = 34.78$, $p < .01$), indicating that the students' use of diagrams was markedly lower in solving the second two-object problem when a non-length story context was given, compared to when a story context involving length was provided. An interaction effect between the response type and country was also found ($\chi^2_{(1)} = 26.98$, $p < .01$), pointing to the lower diagram use evidenced by the Japanese students being more pronounced where the one-object problems were concerned.

Table 3
Percentages of correct answers from the Japanese and New Zealand students

Kind of problem	Category of problem	Percentages of correct answers		$\chi^2_{(1)}$ Value
		Japanese Students	New Zealand Students	
Length	One-object	49.66	78.03	27.29**
	Two-object	49.66	64.74	7.46*
Non-length	One-object	50.70	70.00	11.38**
	Two-object	40.85	65.33	17.58**

*$p < .01$; **$p < .001$.

In order to find out whether the differences in the production of correct answers were likely to have resulted from differences in the extent to which diagrams were used, the relationships between correct answers and diagram use were analysed. Table 4 shows the percentages of Japanese and New Zealand students in each of the following categories of providing correct answer with diagram (P + D+), correct answer without diagram (P + D−), incorrect answer with diagram (P − D+), and incorrect answer without diagram (P − D−). Except in the case of two-object non-length problems, chi-test results showed that there were significantly higher percentages of New Zealand students in the P + D+ answer category, indicating that proportionally more of them used diagrams and provided the correct answers to all the problems given (except the two-object non-length problems). In contrast, significantly higher percentages of Japanese students were in the P − D− answer category where the one-object problems are concerned, indicating that proportionally more of them did not use diagrams and provided the incorrect answers to the one-object problems given. As far as the two-object non-length problems are concerned, the results also showed that significantly higher proportions of the New Zealand students were able to correctly solve them without the use of diagrams (P + D−), and significantly higher proportions of the Japanese students used diagrams in attempting to solve them but got the answer wrong (P − D+).

Because these analyses were conducted using a large sample size, effect size independent of the sample size was calculated using the method suggested by Sugisawa (1999). This method employs the following formula to calculate effect size when a chi-test has been used (note that N = the sample size).

$$\omega = \sqrt{\frac{\chi^2}{N}}$$

Except for the result relating to the two-object length problem in the P + D+ category (Table 4), all of other results indicated as significant in Tables 2 and 3 were confirmed to exceed at least the criteria for small effects detailed in Cohen (1992).

As noted earlier, analyses were also carried out to check whether there were any differences in the qualities of diagrams produced by students from the two countries. Table 5 shows the percentages of high-quality diagrams produced by the students according to their respective countries. Only one significant difference was found: with the one-object problems with length story context, the New Zealand students produced a significantly higher percentage of what were deemed to be high-quality diagrams. No other significant differences were found − which means that, where the other problems were concerned, the percentages of high-quality diagrams produced by the students from the two countries can be considered equivalent.

Table 4
Percentages of Japanese and New Zealand students in each of the answer categories

Answer category	Kind of problem	Category of problem	Percentage of students		$\chi^2_{(1)}$ Value
			Japan	New Zealand	
P + D+	Length	One-object	18.79	49.13	32.35***
		Two-object	31.54	42.20	3.89*
	Non-length	One-object	21.13	38.67	10.66**
		Two-object	7.75	12.00	1.48
P + D−	Length	One-object	30.87	28.90	0.15
		Two-object	18.12	22.54	0.96
	Non-length	One-object	29.56	31.33	0.11
		Two-object	33.10	53.33	12.15***
P − D+	Length	One-object	18.79	11.56	3.30
		Two-object	25.50	16.76	3.71
	Non-length	One-object	9.86	9.33	0.02
		Two-object	18.31	4.00	15.31***
P − D−	Length	One-object	31.54	10.40	22.20***
		Two-object	24.83	18.50	1.91
	Non-length	One-object	38.03	20.67	10.65**
		Two-object	40.85	30.67	3.30

*$p < .05$; **$p < .01$; ***$p < .001$.

Table 5
Percentages of high-quality diagrams in the Japanese and New Zealand groups

Story context	Problem type	Percentages of high-quality diagrams		$\chi^2_{(1)}$ Value
		Japan	New Zealand	
Length version	One-object	60.71	80.95	7.76*
	Two-object	69.41	75.49	0.86
Non-length version	One-object	65.91	80.56	3.12
	Two-object	29.73	41.67	0.92

*$p < .01$.

The relationship between diagram classification and correct solutions provided for the problems was also examined to check the validity of the classification used. With the use of chi-test analyses, it was found that the diagrams classified as ‘‘high quality’’ significantly produced more correct answers for the problems given compared to those classified as ‘‘low quality’’ in both one-object problems (length version: $\chi^2_{(1)} = 41.80$, $p < .01$; non-length version: $\chi^2_{(1)} = 6.28$, $p < .01$) and two-object problems (length version: $\chi^2_{(1)} = 25.43$, $p < .01$; non-length version: $\chi^2_{(1)} = 14.33$, $p < .01$). These findings confirmed the validity of the classification method employed.

3.2. Student views and daily learning behaviors

Table 6 shows the means and standard deviations of the students' responses to the questionnaire items administered. For all items, the means and standard deviations were confirmed not to be distorted. Using *t*-tests, comparisons were made between the mean scores of students in each country on each of the items, and the outcomes are also shown in Table 6.

In their daily learning behaviors, the New Zealand students evidenced significantly higher means in their reported use of diagrams in solving math word problems, as well as in trying to use the kinds of diagrams shown by their teacher to solve other similar math problems. In contrast, the Japanese students evidenced significantly higher means in their reported attention to the diagram use shown by their teacher on the board, and in attempting to copy the way their teacher used diagrams.

In their appraisal of whether the use of diagrams helps figure out how to solve math word problems, the Japanese students were found to have a significantly higher mean. However, the Japanese and New Zealand students did not differ on the other items assessing the students' views about the usefulness of diagrams.

The Japanese students also evidenced significantly higher means on the questionnaire items indicating experience of difficulty in diagram use, while the New Zealand students had higher means on those items indicating experience of ease and greater confidence in diagram use.

Where the students' perception of their teacher's behavior is concerned, the Japanese students evidenced a significantly higher mean in reporting their teacher's use of diagrams to explain how to solve math word problems. However, the New Zealand students had significantly higher means on items indicating encouragement by their teacher to use diagrams, and teacher's demonstration to the class of how to use diagrams in solving math word problems.

An effect size analysis confirmed that the significant results obtained here all exceeded Cohen's (1992) criteria. Here, the following formula was used (where n_1 is the sample size of the first group, n_2 is the sample size of the second group, and t is the absolute value of t)

$$d = |t| \times \sqrt{\frac{n_1 + n_2}{n_1 n_2}}.$$

3.3. Relationship between questionnaire responses and actual use of diagrams

Table 7 shows the correlation values between the students' responses to questionnaire items about their daily learning behaviors and perceptions, and their actual use of diagrams. Students were categorized as either ‘‘using diagrams’’ (which was scored 1) or ‘‘not using diagrams’’ (which was scored 0) according to whether they used at least one diagram in attempting to solve the problems they were given. Correlation values were calculated using this categorization and the students' responses to the questionnaire items, for both countries combined and separately.

Table 6
Means and standard deviations of students' responses to questionnaire items

Item	Japan		New Zealand		$t_{(n-1)}$ Value
	Mean	(SD)	Mean	(SD)	
Daily learning behaviors					
Do you usually use diagrams in solving math word problems?	2.70	(1.13)	2.93	(1.10)	2.59**
Do you try to use the kinds of diagrams shown by your teacher to solve other similar math problems?	2.99	(1.16)	3.42	(1.08)	4.74***
Do you try to copy the way your teacher uses diagrams to solve math word problems?	3.86	(1.33)	3.17	1.22	−6.76***
Do you pay attention to the use of diagrams for solving math word problems that your teacher shows on the board during class?	3.90	(0.99)	3.56	1.13	−4.00***
Do you try to use the kinds of diagrams shown in your textbooks to solve other similar math problems?	2.93	(1.11)	2.96	1.12	0.31
Views about the usefulness of diagrams					
Do you think the use of diagrams is helpful in efficiently solving math word problems?	3.95	(1.02)	3.81	(1.02)	−1.62
Do you think it is good to use diagrams in solving math word problems?	3.88	(1.01)	3.95	(1.01)	0.79
Do you think the use of diagrams helps you figure out how to solve math word problems?	3.74	(1.02)	3.49	(1.17)	−2.86**
Perception of the difficulty of diagram use					
In general, do you know how to construct diagrams for solving math word problems?	3.02	(1.03)	3.70	(0.98)	8.40***
How troublesome is it for you to use diagrams in solving math word problems?	3.01	(1.05)	2.26	(1.03)	−8.93***
How difficult is it for you to make diagrams by yourself for solving math word problems?	3.17	(1.04)	2.43	(1.08)	−8.62***
How easy is it for you to use diagrams in solving math word problems?	3.19	(0.85)	3.85	(0.92)	9.21***
Do you know what kinds of diagrams are helpful in solving different kinds of math word problems?	2.72	(1.06)	3.37	(1.07)	7.49***
Perception of teacher's behaviors					
Do your math teachers use diagrams to explain how to solve math word problems?	3.57	(1.11)	3.34	1.10	−2.56*
Do you think your math teachers use diagrams to efficiently solve math word problems?	3.58	(1.09)	3.50	1.19	−0.86
Do the diagrams that your math teacher uses to show how to solve math word problems help you to understand how those problems can be solved?	3.70	(1.03)	3.67	1.20	−0.31
Are you told or encouraged by your math teacher to use diagrams in solving math word problems?	2.67	(1.20)	3.04	1.19	3.81***
Does your math teacher teach your class how to use diagrams in solving math word problems?	2.98	(1.08)	3.22	1.15	2.70**

*$p < .05$; **$p < .01$; ***$p < .001$.

Table 7 shows that using diagrams in daily learning activities, and attempting to use the kinds of diagrams demonstrated by their teacher and those shown in their textbooks to solve other similar math problems, appear related to actual use of diagrams – although the data from the New Zealand students as a group on its own failed to show significant correlations in the latter two items. The students' views about the usefulness of diagrams were also found to significantly correlate with their actual use of diagrams.

Significant *negative* correlations were found – both for the student groups combined and according to their respective countries – between the students' views about the "troublesome" nature of using diagrams and their actual use of diagrams. In contrast, a significant *positive* correlation was found between the students' (combined) responses to "In general, do you know how to construct diagrams for solving math word problems?" and their actual diagram use.

For the Japanese students, reported teacher instruction or encouragement to use diagrams was also significantly correlated with actual diagram use. On the other hand, a significant negative correlation was found between the New Zealand students' report of their teachers' use of diagrams in explaining how to solve math word problems and the students' actual use of diagrams.

Analysis of effect size was again undertaken, this time using $|r|$ (the absolute value of r) to calculate the effect size, based on Sugisawa's (1999) suggestion. The result revealed that, except for the significant negative correlation found

Table 7
Correlations between the students' responses to the questionnaire items and their actual use of diagrams in solving the problems given

Item	Correlation value, *r*		
	Japan	NZ	Combined
Daily learning behaviors			
Do you usually use diagrams in solving math word problems?	.35***	.30***	.33***
Do you try to use the kinds of diagrams shown by your teacher to solve other similar math problems?	.29***	.07	.19***
Do you try to copy the way your teacher uses diagrams to solve math word problems?	−.01	−.01	−.04
Do you pay attention to the use of diagrams for solving math word problems that your teacher shows on the board during class?	.13	.04	.06
Do you try to use the kinds of diagrams shown in your textbooks to solve other similar math problems?	.17**	.02	.10*
Views about the usefulness of diagrams			
Do you think the use of diagrams is helpful in efficiently solving math word problems?	.18**	.12*	.14***
Do you think it is good to use diagrams in solving math word problems?	.24***	.08	.16***
Do you think the use of diagrams helps you figure out how to solve math word problems?	.12*	.13*	.12**
Perception of the difficulty of diagram use			
In general, do you know how to construct diagrams for solving math word problems?	.05	.09	.10*
How troublesome is it for you to use diagrams in solving math word problems?	−.17**	−.14*	−.17***
How difficult is it for you to make diagrams by yourself for solving math word problems?	−.05	−.06	−.08*
How easy is it for you to use diagrams in solving math word problems?	−.00	.07	.07
Do you know what kinds of diagrams are helpful in solving different kinds of math word problems?	.05	.01	.05
Perception of teacher's behaviors			
Do your math teachers use diagrams to explain how to solve math word problems?	.06	−.12*	−.04
Do you think your math teachers use diagrams to efficiently solve math word problems?	.08	−.10	−.02
Do the diagrams that your math teacher uses to show how to solve math word problems help you to understand how those problems can be solved?	.10	−.07	.01
Are you told or encouraged by your math teacher to use diagrams in solving math word problems?	.16**	−.03	.08
Does your math teacher teach your class how to use diagrams in solving math word problems?	.07	−.03	.03

*$p < .05$; **$p < .01$; ***$p < .001$.

with the combined data on perception of difficulty in making one's own diagrams for problem solving ($r = -.08$), all the other significant correlations obtained and shown in Table 7 exceeded Cohen's (1992) criteria.

4. Discussion

The results of this study confirm initial predictions that on average the New Zealand students would evidence better performance in math word problem solving compared to their Japanese counterparts. What is more important, however, is the finding that significantly higher percentages of the New Zealand students used diagrams in their problem solving *and* obtained the correct answer. In contrast, significantly higher percentages of the Japanese students did *not* use diagrams in their problem solving *and* produced incorrect answers. Although using diagrams in no way guarantees the production of correct answers (as the P − D+ cases in the present study show), the above findings do add further supporting evidence to the notion that using diagrams in math word problem solving is largely beneficial (e.g., Cheng, 2002; Cox, 1999; Koedinger & Terao, 2002). They also lend support to Ichikawa's (1993, 2000) earlier observations of problematically low levels of spontaneous diagram use among Japanese students in solving math word problems. The reasons that contribute to this low diagram use are important to know.

4.1. Reasons for lower levels of diagram use

The results from the analysis of the students' responses to the questionnaire items indicate that the Japanese and New Zealand student groups did not differ much in their appreciation of the usefulness of diagrams: the two groups differed in only one of the three questionnaire items, and the Japanese group evidenced the higher score. This appreciation of the usefulness of diagrams in problem solving was confirmed in the subsequent correlational analysis undertaken as being significantly linked to actual diagram use in students — which is in line with the findings of other authors such as McCombs (1988), Sato (1998), and Uesaka (2002). However, because the Japanese and New Zealand students were more or less equivalent in the extent to which they considered diagrams useful (in fact, if anything, the Japanese students were more appreciative of some of the benefits associated with diagram use), it is unlikely that this factor explains the lower diagram use and poorer performance of the Japanese cohort.

An important group of questionnaire items which the Japanese and New Zealand students significantly differed in their response averages *and* which proved significantly correlated to the combined data of the Japanese and New Zealand students' actual use of diagrams were those relating to perceptions of difficulty in using diagrams. The Japanese students evidenced a significantly lower score in their reports of generally knowing how to construct diagrams for solving math word problems, and significantly higher scores where the troublesome nature of using diagrams, and difficulty experienced in constructing appropriate diagrams by themselves, were concerned. The first of these was found to be significantly correlated, and the latter two significantly negatively correlated, with the students' spontaneous use of diagrams in attempting to solve the problems they were given. Hence, this suggests that one of the reasons for the lower spontaneous use of diagrams among the Japanese students was their perceived difficulties with – or lack of confidence in – the use of such diagrams. This finding adds support to the previously mentioned observation of other authors that self-efficacy (Pintrich & De Groot, 1990; Zimmerman & Martinez-Pons, 1990) and the perception of strategy cost (McCombs, 1988; Sato, 1998) are factors that appear to impact on decisions regarding strategy use.

The findings of this study also provide some clues as to why the Japanese students may perceive greater difficulties in the use of diagrams. Compared to the New Zealand students, they appear to have a tendency to view the use of diagrams as a 'teacher strategy' – to be demonstrated to them – rather than a strategy they should 'own' and use. Their mean score on the item "Do you try to use the kinds of diagrams shown by your teacher to solve other similar math problems?" was significantly lower compared to the New Zealand students' mean score. For the combined and the Japanese data, the responses to this item were another one of those found to significantly correlate with actual diagram use: hence, put another way, the less effort they put to extending the use of diagrams beyond those demonstrated by the teacher, the less likely they are to actually use diagrams when confronted with math word problems to solve.

The view that, as a group, the Japanese students appear to hold – that the use of diagrams is a teacher strategy – is further highlighted by their significantly higher scores on the questionnaire items about trying to copy the way their teachers use diagrams and paying attention to the use of diagrams demonstrated by their teachers on the board during class (neither of which were correlated with actual use of diagrams). These suggest a greater expectation on the part of the students that their teachers will always show them what to do, and is congruent with Purdie and Hattie's (1996) observation that Japanese students were lower in their use of self-regulated learning strategies compared to their Australian counterparts.

4.2. How students' spontaneous use of diagrams might be promoted

A final question that needs to be asked is whether there are any indications in the findings of the present study as to how the use of diagrams in math word problem solving could be promoted. Firstly, for the Japanese students in particular, there are indications that their teachers need to spend more time in class teaching students how to actually use diagrams: although the Japanese cohort scored higher in their report of their teacher using diagrams to explain how to solve math word problems (congruent with the results of TIMSS 2003, reported by Mullis et al., 2004), they evidenced a lower score on the item about their teacher teaching their class *how to* actually use diagrams. One important technique employed in teaching students how to construct and use diagrams is to create opportunities for students to appreciate their use as communication tools, as suggested in the New Zealand curriculum (New Zealand Ministry of Education, 1992). If teachers create situations in which students need to use diagrams in explanations they provide, it will likely improve the students' understanding of how to use diagrams and deepen their understanding of the materials being learnt. Creating more chances for students to learn *how to* use diagrams will not only improve the students' understanding of and efficacy in diagram use, but also develop confidence and reduce expectations of encountering difficulties.

Secondly, teachers should explicitly encourage students to use diagrams as tools of problem solving. In the present study, the Japanese students manifested a significantly lower score on the questionnaire item asking whether they are told or encouraged by their teachers to use diagrams explicitly. The Japanese students' responses to this item were subsequently found to be significantly correlated with their actual diagram use – suggesting that the Japanese students were more likely to spontaneously use diagrams if they had previously received encouragement from their teachers to do so. It would therefore appear important for Japanese teachers to not only take more time to teach students how to construct and use diagrams by themselves, but also to provide greater explicit encouragement to their students to use diagrams.

The findings of the present study confirm previous observations that it is necessary for students to appreciate the value of using diagrams in solving math word problems if they are to use them. The findings further point to confidence and perceived efficacy in using diagrams as additional necessary ingredients for students to be predisposed to spontaneously use diagrams when confronted with math word problems to solve. One important limitation of the present study is that, although they were confirmed to be significant, the correlation values obtained and which point to these conclusions are quite low in magnitude. It would therefore be useful in future studies to explore further (with perhaps the use of different approaches) the effects of factors such as confidence and perceived efficacy in students' spontaneous use of strategies, including the use of diagrams for solving math word problems. Perhaps more importantly, however, future studies need to investigate whether an implementation of teaching strategies such as those suggested here would produce the desired outcomes. The results of such investigations would have potentially wide applications as, although the New Zealand students generally evidenced greater diagram use compared to their Japanese counterparts, the overall proportion of students not using diagrams was highest in *both* countries. Furthermore, the New Zealand students were found to produce a significantly higher percentage of high-quality diagrams only where the one-object problems with length story context were concerned – and these could be considered as being the easiest of the problems given. These findings therefore point to a clear need for the development of students' skills in using diagrams to help solve the full range of math word problems they could encounter.

Acknowledgements

The authors would like to express their gratitude to Howard Gilbert, Tomokazu Haebara, Drew Hayward, Ken Hirakawa, Miwa Inuzuka, Kazuaki Ishida, Yoshio Kaburagi, Makoto Miyashita, Yoshinori Oyama, Mark Phillips, Colin Prentice, Murray Richardson, and Trevor Wilson for their helpful comments and advice, and/or for their assistance in the collection and analysis of the data. This study was supported by a grant from the Center for Research of Core Academic Competences, School of Education, The University of Tokyo, Japan.

References

Ainsworth, S., & Th Loizou, A. (2003). The effects of self-explaining when learning with text or diagrams. *Cognitive Science, 27*, 669–681.

Cheng, P. C. H. (2002). Electrifying diagrams for learning: principles for complex representational systems. *Cognitive Science, 26*, 685–736.

Cheng, P. C. H. (2004). Why diagrams are (sometimes) six times easier than words: benefit beyond locational indexing. In A. Blackwell, K. Marriott, & A. Shimojima (Eds.), *Diagrammatic representation and inference, third international conference, diagrams 2004, LNAI 2980* (pp. 242–254). Heidelberg: Springer.

Cohen, J. (1992). A power primer. *Psychological Bulletin, 112*, 155–159.

Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive apprenticeship: teaching the crafts of reading, writing and mathematics. In L. B. Resnick (Ed.), *Knowing, learning and instruction* (pp. 453–494). Hillsdale, NJ: Lawrence Erlbaum Associates.

Cox, R. (1999). Representation construction, externalized cognition and individual differences. *Learning and Instruction, 9*, 343–363.

De Bock, D., Verschaffel, L., Janssens, D., Van Dooren, W., & Claes, K. (2003). Do realistic contexts and graphical representations always have a beneficial impact on students' performance? Negative evidence from a study on modelling non-linear geometry problems. *Learning and Instruction, 13*, 441–463.

De Corte, E., Greer, B., & Verschaffel, L. (1996). Psychology of mathematics teaching and learning. In D. C. Berliner, & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 491–549). New York: Macmillan.

Dufour-Janiver, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 110–120). Hillsdale, NJ: Erlbaum.

Hall, R., Kibler, D., Wenger, E., & Truxaw, C. (1989). Exploring the episodic structure of algebra story problem-solving. *Cognition and Instruction, 6*, 223–283.

Hembree, R. (1992). Experiments and relational studies in problem-solving – a metaanalysis. *Journal for Research in Mathematics Education, 23*, 242–273.

Ichikawa, S. (1993). Suugakutekina kangaekatawo megutteno soudanto sidou [Case report of cognitive counseling in mathematical thinking]. In S. Ichikawa (Ed.), *Gakusyuuwo sasaeru nintikaunsering: Shinrigakuto kyouikuno aratanasetten* [*Cognitive counseling that supports learning: A new approach bridging psychology and education*] (pp. 36–61). Tokyo: Brain Press.

Ichikawa, S. (2000). *Benkyouhouga kawaru hon* [*A book about changing the approach to learning*]. Tokyo: Iwanami Press.

Japanese Ministry of Education. (1998). *Tyuugakakukou gakusyuu sidouyouryou* [*National curriculum standards for junior high school*]. Tokyo: Japanese Government Printing Office.

Koedinger, K. R., & Terao, A. (2002). A cognitive task analysis of using pictures to support pre-algebraic reasoning. In C. D. Schunn, & W. Gray (Eds.), *Proceedings of the twenty-fourth annual conference of the Cognitive Science Society* (pp. 542–547). Mahwah, NJ: Lawrence Erlbaum Associates.

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*, 65–99.

Mayer, R. E. (1981). Frequency norms and structural-analysis of algebra story problems into families, categories, and templates. *Instructional Science, 10*, 135–175.

Mayer, R. E. (2003). The promise of multimedia learning: using the same instructional design methods across different media. *Learning and Instruction, 13*, 125–139.

McCombs, B. L. (1988). Motivational skill training: affective learning strategies. In C. E. Westin, E. T. Goetz, & A. Alexander (Eds.), *Learning and study strategies: Issues in assessment, instruction, and evaluation* (pp. 141–169). San Diego: Academic Press.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Nagasaki, E. (Ed.). (2005). *PISA2003nentyousa, TIMSS2003nentyousa: Sansuu, suugakunikansuru hyouka, bunseki repouto* [*PISA2003, TIMSS2003: Report on the evaluation and the analysis of mathematics*]. Tokyo: National Institute for Educational Policy Research of Japan.

National Council of Teachers of Mathematics. (2000). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

New Zealand Ministry of Education. (1992). *Mathematics in the New Zealand curriculum*. Wellington, New Zealand: Learning Media.

New Zealand Ministry of Education. (2005). Deciles information. Available from. http://www.minedu.govt.nz/index.cfm?layout=document&documentid=7693. Accessed 18.01.06.

Organisation for Economic Co-operation and Development. (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris, VA: Author.

Pedone, R., Hummel, J. E., & Holyoak, K. J. (2001). The use of diagrams in analogical problem solving. *Memory & Cognition, 29*, 214–221.

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology, 82*, 33–40.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Purdie, N., & Hattie, J. (1996). Cultural differences in the use of strategies for self-regulated learning. *American Educational Research Journal, 33*, 845–871.

Reed, S. K. (1999). *Word problems: Research and curriculum reform*. Mahwah, NJ: Erlbaum.

Sato, J. (1998). Effects of learners' perception of utility and costs, and learning strategy preferences. *Japanese Educational Psychology, 46*, 367–376.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. San Diego: Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Stern, E., Aprea, C., & Ebner, H. G. (2003). Improving cross-content transfer in text processing by means of active graphical representation. *Learning and Instruction, 13*, 191–203.

Sugisawa, T. (1999). Statistical power of educational psychology research in Japan. *Japanese Journal of Educational Psychology, 47*, 150–159.

Uesaka, Y. (2002). Suugakuniokeru zuhyouno yuukouseito konnanseino ninchi [The relationship between students' use, and beliefs about the effectiveness and difficulties involved in the use of diagrams]. *Proceedings of the 44th annual conference of Japanese educational psychology*. Tokyo: The Japanese Association of Educational Psychology. p. 176.

Uesaka, Y. (2003). Zuhyousakuseino sikakukasukiruga suugakutekimondaikaiketuni oyobosu eikyou [The influence of visualized skills on diagram use in mathematical problem solving]. *Proceedings of the 67th annual conference of the Japanese Psychological Association*. Tokyo: The Japanese Association of Educational Psychology. p. 501.

Van Essen, G., & Hamaker, C. (1990). Using self-generated drawing to solve arithmetic word-problems. *Journal of Educational Research, 83*, 301–312.

Yoshida, H. (1991). *Kodomoha kazuwo donoyouni rikai siteirunoka - kazoerukotokara bunsuumade* [*How do children understand numbers? From counting to fractions*]. Tokyo: Shinyousha.

Zimmerman, B. J., & Martinez-Pons, M. (1990). Student differences in self-regulated learning: relating grade, sex, and giftedness to self-efficacy and strategy use. *Journal of Educational Psychology, 82*, 51–59.