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Research in Perspective

Skills underlying mathematics: The role of executive function in the development of mathematics proficiency

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ABSTRACT

The successful learning and performance of mathematics relies on a range of individual, social and educational factors. Recent research suggests that executive function skills, which include monitoring and manipulating information in mind (working memory), suppressing distracting information and unwanted responses (inhibition) and flexible thinking (shifting), play a critical role in the development of mathematics proficiency. This paper reviews the literature to assess concurrent relationships between mathematics and executive function skills, the role of executive function skills in the performance of mathematical calculations, and how executive function skills support the acquisition of new mathematics knowledge. In doing so, we highlight key theoretical issues within the field and identify future avenues for research.

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1. Introduction

Children's underachievement in mathematics is a consistent and significant problem [33] with 21% of 11-year-olds leaving primary school without reaching the mathematics level expected of them, and 5% failing even to achieve the numeracy skills expected of a 7-year-old [45]. These problems endure into adulthood, and it is estimated that a fifth of adults have numeracy skills below the basic level needed for everyday situations [95]. Mathematics ability is crucial for success in Western societies [3] and poor mathematics skills have a bigger impact on life chances than poor literacy [75]. Given the significant economical and societal impact of these problems it is important to understand in detail the processes involved in learning and performing mathematics.

Many factors contribute to differences in mathematics achievement, including attitudes [66], motivation [87], language ability [31] and IQ [67], in addition to social [18], and educational factors [72,74]. It is clear that domain-specific numerical skills and knowledge are important for success with mathematics [43,57], but other cognitive factors also play an important role. In particular, the domain-general skill of holding and manipulating information in mind (working memory) has been found to be critical [76]. Inhibition, the ability to suppress distracting information and unwanted responses [16,40,58,84], and shifting, the ability to flexibly switch attention between different tasks [98], have also been implicated in mathematics achievement. These processes fall under the umbrella of executive function (EF); skills required to monitor and control thought and action (Text box 1). Within the research literature, four types of study have aimed to describe and understand the role that EF skills

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Text box 1–What is executive function?

Executive function (EF) is the name given to the group of processes that allow us to respond flexibly to our environment and engage in deliberate, goal-directed, thought and action. Executive function forms the basis of abilities such as problem solving and flexible thinking and is most likely to be used in the absence of external guidance or when a situation is novel. The study of executive function originated from observations of adults with damage to the frontal lobe of the brain and the study of these patients has led to a very strong link in research between executive processes and frontal lobe function. Executive function skills begin to emerge in infancy [30] but are among the last cognitive abilities to mature, continuing to develop into late adolescence [25,53,65].

The three EF skills most commonly studied, particularly within the developmental literature, are *inhibition*: suppressing distracting information and unwanted responses, *shifting*: flexibly switching between different tasks, and *updating* or *working memory*: monitoring and manipulating information in mind. The majority of studies addressing the role of working memory in mathematics are based on the working memory model of Baddeley and Hitch [8,9]. This comprises an attentional control system (the central executive), supported by two subsidiary slave systems for the short-term storage of verbal and visuospatial information (the phonological loop and visuospatial sketchpad, respectively).

play in supporting mathematics achievement: Correlational studies examining the relationship between mathematics and EF at a single time point; experimental studies exploring the role of EF in performing mathematical calculations, learning or training studies that aim to pinpoint how EF skills support the acquisition of new mathematics knowledge and neuroimaging studies, revealing the neural mechanisms by which EF supports mathematics (Text box 2). Here we review this literature with the aims of synthesising current knowledge and identifying questions for future research.

2. Correlational studies

The majority of studies investigating the role of EF skills in mathematics have used a cross-sectional correlational design. Using a range of measures these studies have demonstrated that working memory accounts for unique variance in written and verbal calculation, as well as mathematical word problems, across a range of different age groups [1,4,5,11,16,20,47,58,60,61,68,71,82,83,90,88,92,99]. Importantly it is the ability to manipulate and update, rather than simply maintain, information in working memory that seems to be critical for mathematics proficiency. This variance cannot be explained by other factors such as age, IQ, mathematics ability, processing speed, reading and language skills ([4,11]; but see [36]).

Further evidence that working memory is important for mathematics comes from children who demonstrate a specific difficulty with mathematics. Two recent meta-analyses have suggested that children with mathematics disabilities have particular difficulty with the central executive component of working memory [26,91], especially when numerical information is involved [6,26]. This highlights an issue with many correlational studies where EF tasks involving numerical stimuli are used, e.g. digit span, a verbal working memory task where a string of numbers have to be recalled forwards or backwards. These measures may overestimate the role of EF skills in mathematics compared to non-numerical tasks because of their domain-specificity [76]. However, the fact that working memory predicts mathematics performance even when non-numeric stimuli (e.g. letters or words) are used demonstrates that this is not the sole determinant of the relationship.

Text box 2–: *EF skills and mathematics in the developing brain*

Neuroimaging methods, in particular functional Magnetic Resonance Imaging (fMRI), have opened up the possibility for researchers to determine how individuals of different ages approach numerical processing. In studies which require children and adults to choose the larger of two digits or sets of dots, or verify simple sums, adults typically show more activity than children in posterior parietal areas of the brain, while children show greater activity in frontal areas such as the medial and inferior frontal gyri [7,21,78]. These developmental changes have been interpreted as increased functional specialisation in the parietal brain areas that support numerical cognition alongside decreased dependence on working memory and attention.

Two more recent studies have investigated changes within childhood, either between 7 and 9 years of age [80], or between children of the same age classified as 'counters' or more sophisticated 'retrievers' when solving sums [23]. In contrast to previous work, both of these studies found increased activation in some frontal areas for the more advanced group. An increase in ventrolateral prefrontal cortex activity in retrievers versus counters was attributed to cognitive control over retrieval (e.g. the selection of retrieval strategies and inhibition of procedural strategies). Developmental changes from 7 to 9 years were reflected by an increase in activity in left dorsolateral prefrontal cortex, which the authors hypothesised may reflect more precise manipulation of information in working memory in the older children.

The differences between these two sets of studies may be indicative of an increase, followed by a decrease in the recruitment of frontal brain areas in numerical processing with age. Alternatively, it may be attributable to differences in the task being performed: Arithmetic [23,80] may require increased recruitment of frontal brain areas with age or strategy change, whereas frontal areas may be less involved in processing of numeric magnitude with age [7,21,78]. Nevertheless, it remains unclear exactly what the frontal activity across these studies represents, in particular whether it reflects the involvement of EF skills. To demonstrate this conclusively it would be necessary to show that the frontal activity seen in mathematical tasks correlates with activity in the same area on a measure of executive processing.

Fewer studies have investigated the role of inhibition and shifting in mathematics performance and the findings are mixed. The majority of studies suggest that inhibitory control abilities do predict performance in mathematics ([14,16,34,40,58,61,84,92]; but see [5]). A recent meta-analysis demonstrated that shifting ability does predict performance in mathematics [98], however it remains unclear whether shifting is an independent predictor of mathematics over and above general intelligence.

Correlational studies provide convincing evidence of a relationship between EF skills and mathematics, which may be stronger than the relationship between EF skills and other areas of academic performance (Text box 3). The majority of studies have used standardised tests of mathematics which confound factual (e.g. $6+4=10$), conceptual (e.g. knowing that addition is the inverse of subtraction) and procedural (e.g. 'carrying' when adding above 10) knowledge. Yet it is well established that there are complex relationships among these components [10,44]. Furthermore, individuals differ in their profile of performance across these components, and may have strengths in one component but not others [32], suggesting that different mathematics components may rely on differential sets of EF skills. Indeed, there is emerging evidence that the contribution of executive skills may differ across these components. Hecht et al. [47] showed that while working

Text box 3–Are executive function skills important for learning in general?

Although there is a clear link between executive function skills and mathematics achievement, it is possible that this relationship is not specific and that EF skills are important for learning and performance across all academic subjects. A number of studies have compared the role of EF skills across different domains. The findings to date suggest that, at the start of school, inhibition and working memory contribute to performance in tests of both mathematics and reading ([86,92,94]; but see [15]). However, there is some evidence that in 5-year-olds, EF skills explain more variance in mathematics than in reading [96]. In later years working memory and inhibition skills predict performance on school exams in English, mathematics and science at both 11 [84] and 14 [73] years of age, and shifting appears to predict both mathematics and reading scores across development (Yeniad et al., [98]). Yet in a comprehensive study that tracked performance every year from kindergarten to grade 5, Geary et al. [39] showed differing relationships between executive working memory and mathematics and reading over time. While the importance of working memory for reading decreased with age, the relationship between mathematics and working memory increased. This suggests that while EF skills are important for academic achievement beyond mathematics, the precise relationships with different domains may differ. This necessitates further study beyond simple correlations between EF measures and scores on standardised tests of academic achievement in order to examine the mechanisms and pathways involved.

memory ability was related to fraction computation, it was not a predictor of conceptual understanding of fractions. In contrast, inhibition has been linked to the application of additive concepts [79]. Future research should take a similar componential approach in order to understand how EF skills support different aspects of mathematical competence. We will return to this issue later in *Theoretical implications.*

3. Experimental studies

A small number of studies have made use of experimental methods to pinpoint the precise EF skills involved in the performance of mathematical operations. These studies have utilised dual-task methods that require participants to solve arithmetical problems while they perform a concurrent task with an EF load. Several studies have shown that adults′ arithmetical processing is impaired by dual-tasks involving the manipulation of information within the central executive component of working memory, but not tasks that require only storage of verbal information [27,28,46,64]. However, the majority of these studies involved an arithmetic verification task, which uses different strategies and processes from generating an answer [19]. Therefore this methodology does not accurately assess the contribution of EF skills to mathematical calculation. Moreover, many of these studies failed to distinguish between the different strategies that participants use to solve the problems, such as retrieval, counting or decomposition strategies. As these different strategies involve different combinations of procedural, conceptual and factual knowledge they may create different EF demands. In studies where strategy use has been considered, the effects of working memory load have been greatest when participants use counting, and least for retrieval [52,54,55]. It is critical, therefore, to consider the strategy that participants are using.

The vast majority of studies using dual-task methods have involved adult participants with years of experience of performing arithmetical operations. An individual′s mathematical ability appears to affect their reliance on EF skills to perform arithmetical operations [56]. It is likely, therefore, that the contribution of working memory to mathematical processing will differ for children who are at different stages of learning mathematics. Very few studies have addressed this question with children, although those that have suggest that both children′s symbolic [22,54,69] and non-symbolic [97] mathematics skills rely on working memory. McKenzie et al. [69] showed that 8–9-year-olds′ arithmetic performance was affected by both verbal and visuospatial disruption, while 6–7-year-olds were only affected by visuospatial disruption. This might suggest that younger and older children make use of different types of working memory to solve sums. However, since passive tasks were used it is possible that the younger children were less attentive to the verbal dual task. Further evidence of differential reliance on verbal and visuospatial working memory comes from Caviola et al. [22] who found that, for approximate arithmetic involving carrying procedures, maintaining verbal information impaired performance on horizontally presented sums, while maintaining visuospatial information impaired vertically presented sums. These studies did not compare children′s use of different strategies however. Therefore the differences may have reflected varying reliance on working memory when using the same strategy, or alternative use of different arithmetical strategies.

Imbo and Vandierendonck [54] did consider the strategies used by 10–12-year-olds when they solved arithmetical problems while performing an active concurrent task designed to load the central executive. They found that performance was impaired by the dual task for all strategies that children used, and that this effect was greater for a decomposition strategy than for retrieval or counting. The amount of impairment decreased with age for retrieval and counting but not for decomposition. However, only a restricted age range was considered and it is important to consider how children′s reliance on working memory changes over a greater developmental range. Moreover, as with many studies, this experiment did not include a control dual-task condition and therefore it is not possible to determine whether the observed drop in performance on the arithmetic task is due to the working memory load itself or simply the challenge of performing a dual-task. In a recent study, 9–11-, 12–14-year-olds and adults were asked to solve addition problems by counting, decomposition and retrieval strategies while performing either a concurrent working memory or a control task [42]. Using this more stringent design it was found that loading working memory slowed 9–11-year-olds′s performance on the addition problems for all three strategies, 12–14-year-olds for the two procedural strategies but adults only for counting. This suggests that children do rely on working memory to a greater extent than adults when solving arithmetic problems, most likely due to the fact that all arithmetic strategies are less automatic and efficient in children and therefore rely more on general processing resources.

4. Learning studies

In contrast to experimental methods, which focus on the application of previously acquired mathematics knowledge, a small number of studies have investigated the role of EF skills in learning new mathematical material. Some correlational studies have included a longitudinal component that allows the relationship between EF skills at one time point, and mathematics performance at a later time point, to be examined. On the whole these studies have shown that EF skills do predict mathematics performance in subsequent years [2,13,37,48,49,70,94], however simply measuring performance at a later time point does not necessarily reflect the learning of new mathematical material. A better approach is to predict the growth in mathematics performance between two

time points. Both central executive measures of working memory [39], as well as composite EF measures [24,63], have been found to predict improvements in mathematical competency over a year. Progress in mathematics is also related to improvement in executive working memory [89,93]. These findings suggest that EF skills do play a role in learning new mathematical material in addition to carrying out mathematics operations. This may be limited to some types of mathematical knowledge however. Using a range of mathematics tasks, LeFevre et al. [63] demonstrated that EF predicts growth in acquisition of number facts, but not improvements in procedural or number system knowledge.

An alternative approach to examine the role of EF skills in mathematics learning involves training studies. Three recent studies have attempted to directly train EF skills and link this training to improvements in mathematics. Holmes, Gathercole and Dunning [50] gave 9–10-year-old children with poor working memory skills training using a computerised working memory task. Improvement in mathematics reasoning was seen for the adaptive-training group in a delayed post-test administered 6 months following training. However, there was no comparison group at the delayed post-test and thus it is not possible to link this improvement specifically to the working memory training. Similarly, St Clair-Thompson et al. [85] examined the impact of computerised working memory training in 5- to 8-year-olds. Training did lead to an improvement in working memory skills and mental arithmetic, however no impact on mathematics achievement was observed either at immediate post-test or five months later.

Using a different training method, Kroesbergen et al. [59] employed small group teaching sessions to train working memory in 5-year-olds. They included mainly non-computerised games with either numerical or non-numerical content. Visuospatial working memory improved in both groups compared to controls, but only the numerical training group showed an improvement in counting skills, suggesting that any training needs to be domain-specific. Together these studies demonstrate that while current training programmes improve performance on working memory tests, they have not yet succeeded in training the necessary processes that allow the transfer of this improvement to the classroom. Further study and understanding of the mechanisms and pathways by which EF supports mathematical learning will help to elucidate the processes that need to be trained in order for real-world benefits to be seen.

A final approach, taken by Fuchs et al. [35,38], is to relate the efficacy of mathematics tutoring to children's cognitive skills (including EF skills). They found that attention and working memory measures predicted performance on mathematics measures at the end of training, suggesting that children's EF skills do have an impact on their ability to learn new mathematical material. However, it is difficult to generalise from the findings of these studies since they involved a highly controlled mathematics tutoring program. It remains to be established whether these findings extend to learning mathematics in a classroom situation.

5. Theoretical implications

The majority of current theories of numerical cognition do not incorporate EF processes into their models (e.g. [29,17]). Yet with increasing evidence of the interplay between domain-general and domain-specific skills in the development of mathematics proficiency, it is essential that both are integrated into theoretical frameworks. The Pathways Model of LeFevre et al. [62] addresses this by including linguistic, quantitative and spatial attention pathways to early numeracy and mathematical outcomes. This is a good start, however further specification of the attention pathway is warranted given the liklihood of complex relationships between aspects of EF and different components of mathematical competency.

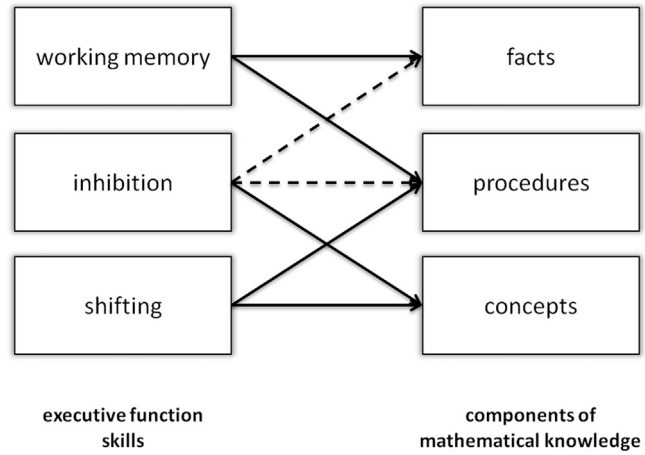

Fig. 1. A theoretical model predicting relationships between executive function skills and components of mathematical knowledge. Dashed lines represent relationships that change over the course of development.

Empirical evidence of these complex relationships is only starting to emerge, neverthless predications can be made on a theoretical basis (Fig. 1). For example, for procedural knowledge, working memory is likely to be important at all ages in order to hold interim answers online while performing other parts of a sum. Inhibition is likely to be especially important at younger ages to suppress less sophisticated strategies, e.g. counting on from the first addend, in order to use more sophisticated strategies, such as counting on from the larger addend. Shifting may also be required to switch between different procedures (e.g. adding and subtracting) when solving complex mathematical problems. For factual knowledge, working memory is likely to play a role in acquiring new facts as both sum and answer need to be held in mind together in order to strengthen the relationship between them. Inhibition may be necessary to suppress answers to related but incorrect number facts, (e.g. inhibit 6 when asked 3×3). This may show an inverse U-shaped function with age, being required most when children have already acquired some number facts but are still learning others. Inhibition, along with shifting, is also likely to be needed when learning new concepts in order to inhibit an automatic procedural approach and shift attention towards the numerical relationships involved (c.f. Siegler and Araya, [81]). Further empirical work is needed to test these predictions and develop stronger theoretical models which accurately represent this complex area of study.

6. Summary

There is now increasing evidence of a strong relationship between EF skills, in particular executive working memory, and children's mathematics achievement. Unfortunately, many practitioners are not explicitly aware of the importance of EF skills for learning [41], suggesting that greater interaction between researchers and practitioners is needed. Moreover, the mechanisms by which EF skills support the acquisition as well as the skilled application of mathematics knowledge are far from clear [12], and this level of understanding is essential in order for this area of research to successfully inform and influence classroom practice. In order to achieve this goal we propose that researchers should move away from the use of standardised mathematics tests towards measures that capture elements of children's factual, conceptual and procedural knowledge. The relationship between EF skills and mathematics in different age groups also needs to be systematically addressed. Both mathematics ability and EF skills

improve during development and therefore the relationship between the two will also change as children get older [51,77]. Only by exploring the differential role of EF skills in multiple components of mathematical knowledge in different age groups, as well as distinguishing between the acquisition and skilled application of this knowledge, will we expose the subtleties in the relationship between EF skills and mathematics, building a theoretical framework within which to interpret findings and generate new testable hypotheses.

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